Minting a Coin

Mik Zlatin

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Intro

This simple to state puzzle is about flipping coins. The twist is that the probability with which the coin lands on heads is also a random variable.

There is a solution to this riddle which involves computation and integrals, but given the solution, you may suspect there is a clever way to do it. And indeed there is.

Riddle

I am minting a random coin. First I pick a value p uniformly at random from the interval $[0, 1]$. Then I mint a coin such that the probability that the coin lands on head is p.

I sell this random coin to Bob for a pretty penny, and Bob flips it 100 times.

Easy Question: What is the expected number of heads?

Riddle Question: Consider the probability that we get k heads. For what value(s) of k is this probability maximized?